SENSITIVITY ANALYSIS BASED ON THE SHARP COMPUTER CODE

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INTRODUCTION

An essential step in modelling applications is the sensitivity analysis to quantify the uncertainty in the model caused by lack of data or uncertain aquifer parameters, stresses, and boundary conditions. Besides the heads, it is also important to know the sensitivity of the intrusion lengths depending on these parameters. Moreover, the knowledge of sensitivity analysis of heads and intrusion lengths can be used during operational management later on.

The model application has been hypothetically made for the Ben Tre aquifer system, which is located on the Southeast of the Mekong Delta (MD) of Vietnam and reportedly, this aquifer, is partly intruded by seawater (see Michael,1971). In a sense of conjunctive use of surface water and groundwater for the regional development prospect, the increase of recharge and a groundwater barrier against seawater intrusion may be necessarily one of alternatives.

One-Dimensional Model

Here we will simulate the one-dimensional confined aquifer case by applying the SHARP model (Essaid, 1990). In the horizontal plane there are three rows of 1250m each and 330 columns of 250m each. The hydro-geological data and aquifer parameters will be as follows:

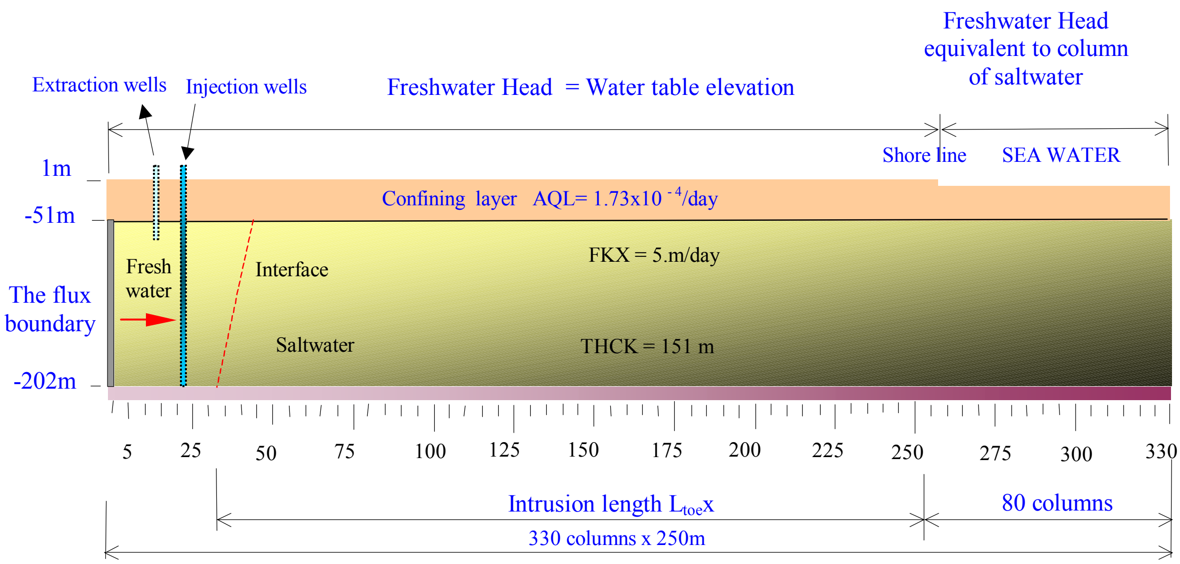


Figure 1 *Cross sectional profile of the model.*

The topmost one is the semi pervious Holocene layer, with C = 5780 days. The permeable Pleistocene layer (main aquifer) has a hydraulic conductivity of 5 m/day and thickness is 151m.

The bottom of the system is considered to be impermeable, i.e. a no-flow boundary. A constant flux boundary is considered at the left boundary, with values varying in the range of 864 m^3/d – 43200 m^3/d. A partly penetrating extraction and a fully penetrating injection wells are located at cells 12 and 14, respectively. The geometry of this problem is illustrated in figure 1.

Two-Dimensional Model

The model consists of a rectangular area in the horizontal plane. Its aquifer system is considered as confined, having the same cross sectional profile as in the one-dimensional model (see the figure 1).

In the horizontal plane view, we simulate the two-dimensional grid system in which there are 15 rows of 1250m each in the y-direction. Grid spacing in the x-direction is 1250m except near the location of wells and the interface where it is reduced to 250m to improve the interface projection accuracy (see Essaid, 1990). Therefore the total columns in x-direction are 146. The salt/fresh interface is considered as an abrupt change of the groundwater density implying no existence of the mixing zone of brackish water. The hydro-geological data and aquifer parameters will be the same as the first case.

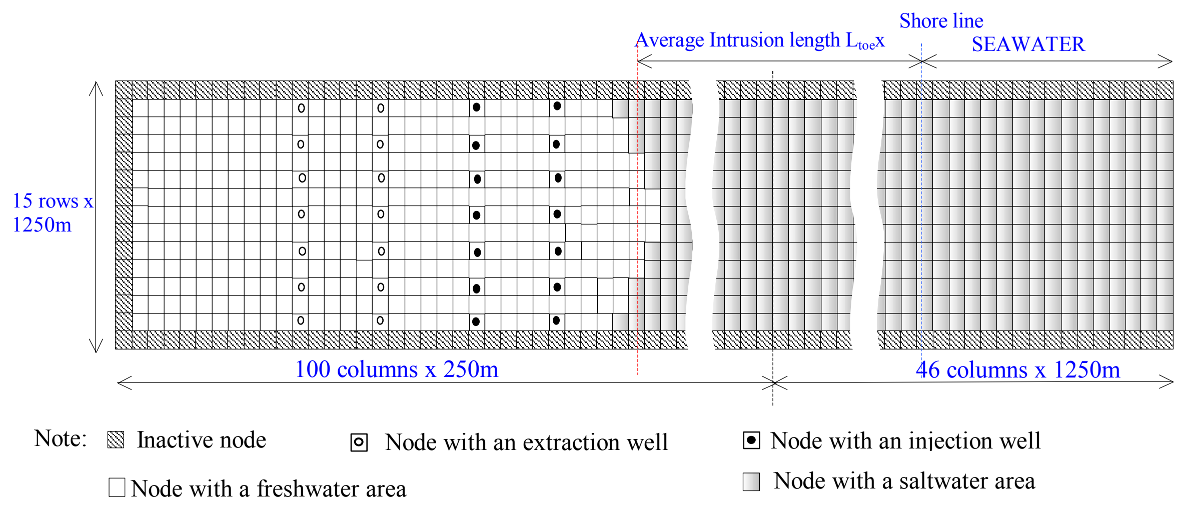


Figure 2 *The horizontal plane view of the aquifer of the two-dimension model*

OBJECTIVES AND PROCEDURES

For the one-dimensional model, the sensitivity analysis is focussed on:

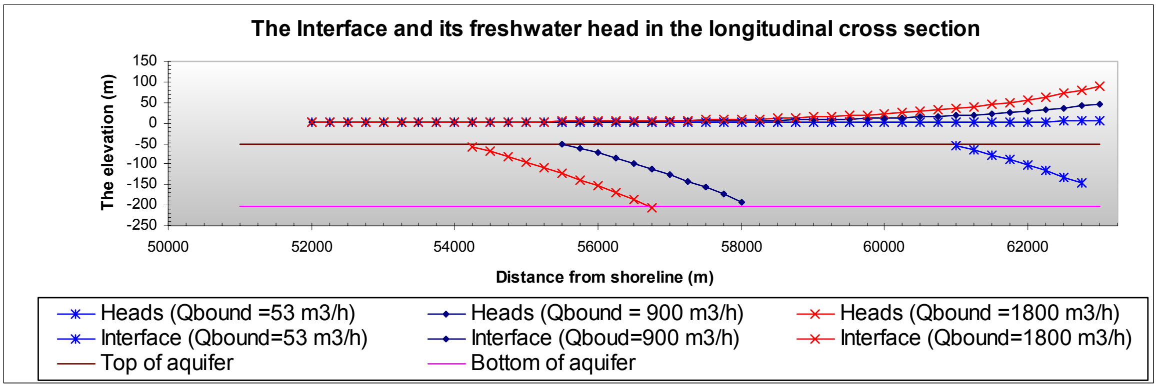
1. Influence of boundary flow rates on the responses of intrusion lengths and freshwater heads;
2. Responses of the intrusion lengths to the injection rates;
3. Responses of the intrusion lengths to the extraction rates;
4. And those with respect to the changes of the aquifer hydraulic conductivity.

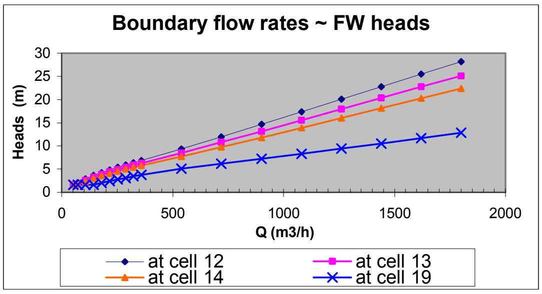
For the two-dimensional model:

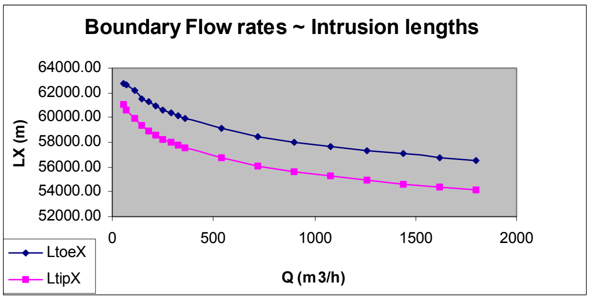
1. The sensitivity analysis aims at understanding of the behaviour of the intrusion length in the horizontal plane. It enables incorporation of the simulation model into the optimization problem by taking the gradients of the response curves of the horizontally averaged intrusion toe length with respect to the injection/extraction rate perturbations.
2. Consequently, the sensitivity analysis here is performed on the responses of the average intrusion toe length to the injection or extraction rates perturbations in turn among a set of 28 active well locations in the model area.

MODEL SIMULATION RESULTS

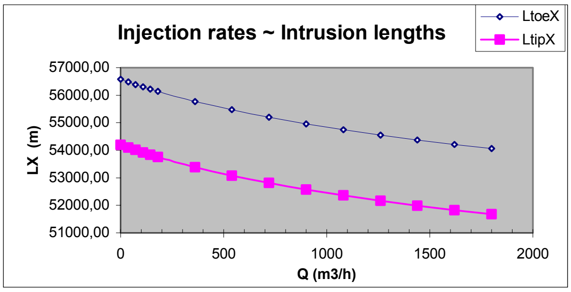
The Flux Boundary Rate Perturbations:

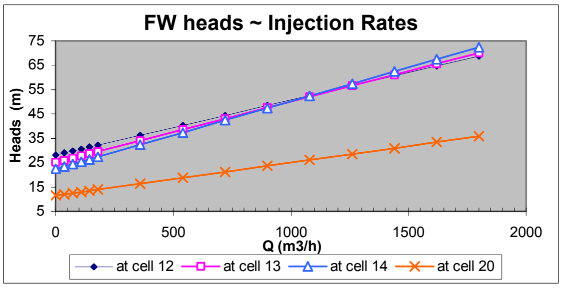


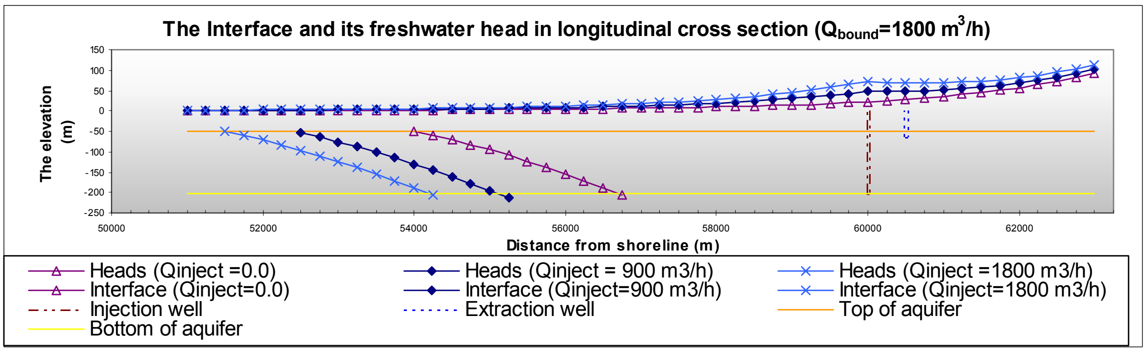




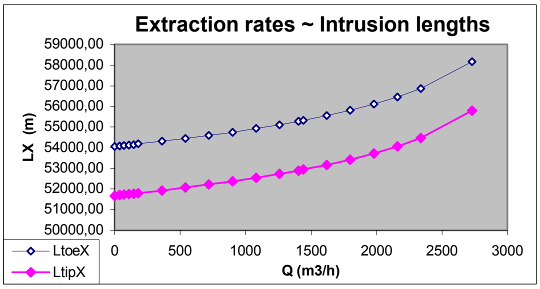
The Injection Rate Perturbations:

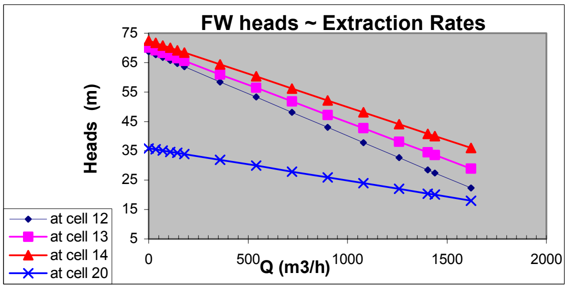


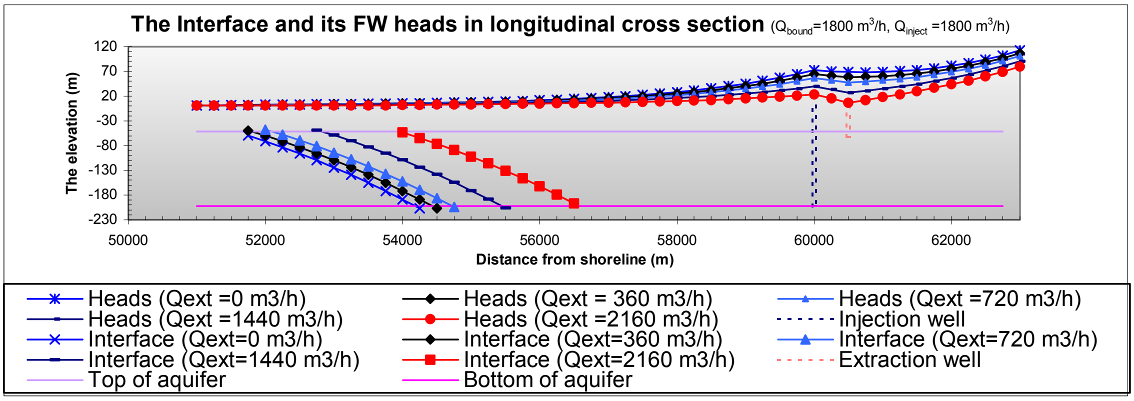




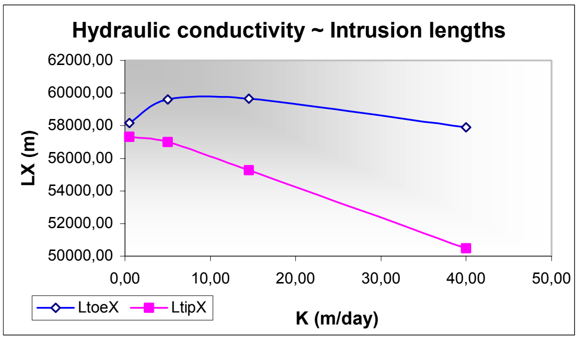
The Extraction Rate Perturbations:

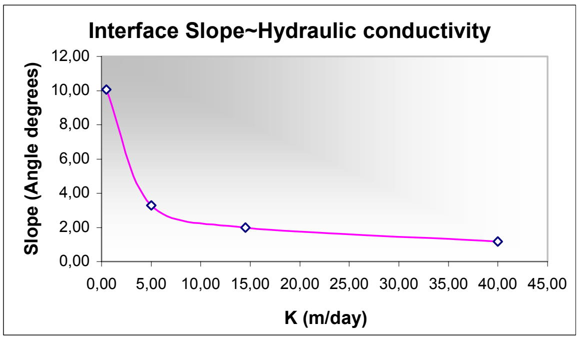


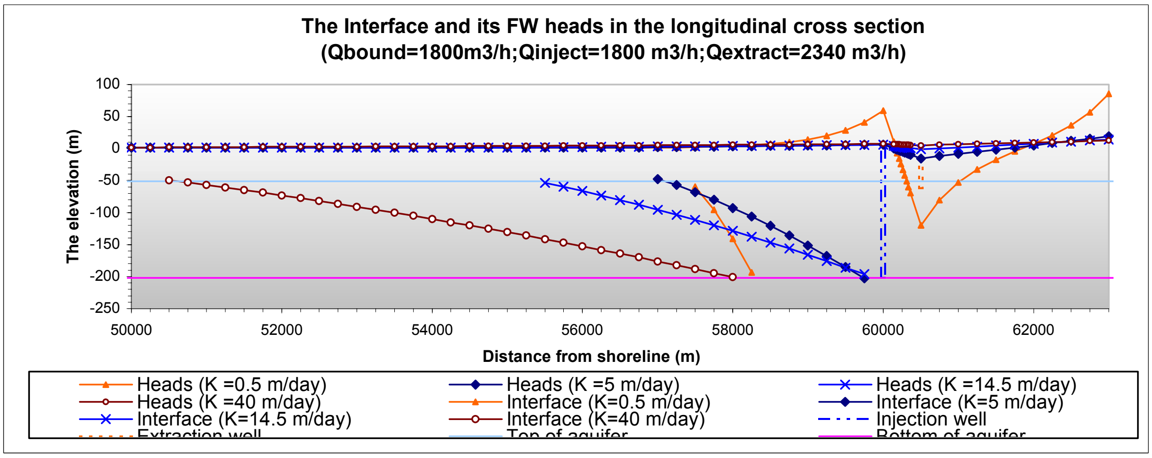




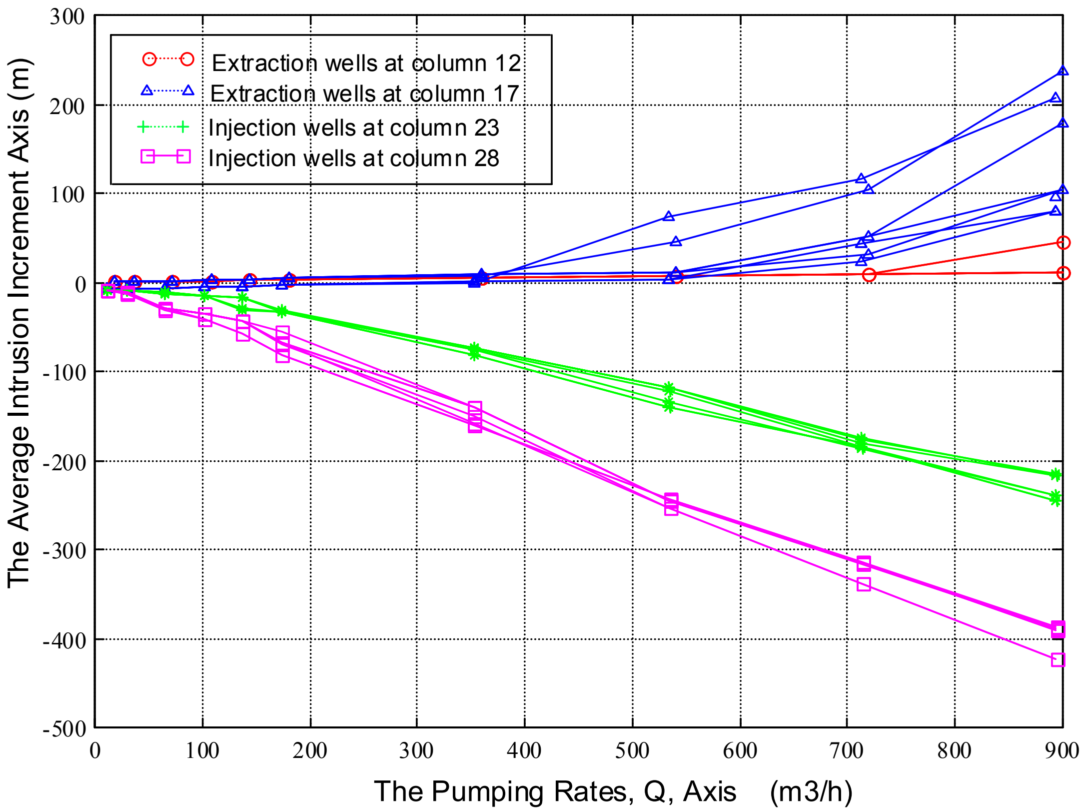
Variations of Hydraulic Conductivity:







The Average Intrusion Increments & Q in The Two-Dimensional Model:



Results:

For the one dimensional case:

- The responses of freshwater (FW) head in a control point are linear with respect to the changes of boundary flow rates, injection rates or extraction rates at a certain well location.
- The movement of the interface (or the intrusion lengths) is non-linear with respect to linear changes of boundary flow rates, injection rates or extraction rates at a certain well location. In case of injection, the gradients of its curve decreases when stress is increasing. For the extraction case, the curvilinear relationship is reverse implying that its tangents will vary from flat to steep.
- The slopes of interfaces of all cases have more or less the same value (3o28' -3o29') when the hydraulic conductivity is kept constant (and equal to 5m/day).
- The interface slope considerably changes from steep to flat if the hydraulic conductivity varies from low to high values. The absolute values of head gradients in cases of low hydraulic conductivity are also greater than the ones in cases of high hydraulic conductivity, especially near the flow boundary or extraction/injection well locations.

For the two-dimensional case:

- The relationship between the average intrusion toe lengths and the extraction/injection rates is also of a non-linear nature.
- At a certain stress value, the closer the location of the well is to the interface, the bigger the gradient of its curve is.
- In a small range of stress value, the response-curved gradients of wells, which are close to the boundaries (in y-direction) are bigger than the ones of wells, which are in the middle in case of injection. It inversely holds for the case of extraction.

DISCUSSIONS & CONCLUSIONS

- In most cases, the model results are quite dependent on the initial freshwater heads, which must be given to each of the injection wells in case the initial interface has to be calculated by the Ghyben-Herzberg formula.
- The results of the sensitivity analysis are incorporatively used for the deterministic linear optimization problem, SEDUMI, (see Ben-Tal et al. 1996), especially for the sequential linearization approach by which the non-linear responses of the average intrusion lengths can be linearized. Using an iterative method (like Newton's method) consecutive points with different gradients on the curves can be found until the solution converges.
- The perturbation size of stress should be chosen as small as possible such that the response increments of the averaged intrusion lengths are still greater than the rounding errors of the computation (see Ahlfeld et al., 2000). This can be achieved using an iterative method to ensure convergence .
- The different responses of the interface with respect to the variation of hydraulic conductivity will be taken into the uncertainty problem, which can hopefully be solved by the quadratic cone (second-order cone) optimization problem (see Ben-Tal et al. (1996), Ndambuki (2001)).

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