EFFECTIVE DISPERSION IN SEAWATER INTRUSION THROUGH HETEROGENEOUS AQUIFERS

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Abstract

Dispersivity is the most significant and difficult parameter to assess in seawater intrusion models. It is well established that dispersion is enhanced by spatial heterogeneity. While real aquifers in general are heterogeneous, most models describing saltwater intrusion into coastal aquifers do not take into account spatial heterogeneity. One of the main objectives of this study is to assess the effect of spatial variability of hydraulic conductivity on effective dispersion in seawater intrusion problems. For this purpose we select Henry’s problem (1964), both in its diffusive and dispersive forms. An anisotropic Henry Problem is defined and the effect of the anisotropy in this type problem is investigated. Spatial heterogeneity is analysed within a geostatistical framework using a methodology based on Monte Carlo analysis. The basic steps are: first, a geostatistical model is selected and a number of heterogeneous hydraulic conductivity fields are generated for each model. Second, Henry’s problem is solved for all heterogeneous fields. Specifically, we aim at finding appropriate effective values for dispersivity as a function of heterogeneity. We find that, for dispersive problems, both toe position (i.e., seawater wedge penetration) and width of the mixing zone can be fairly well reproduced by a homogeneous medium with the same effective hydraulic conductivity and the (constant) dispersion coefficient which is assumed valid at the local scale (local dispersivity coefficient). Results can be improved by slightly increasing the dispersion values, specifically the geometric mean of the longitudinal and transversal dispersivity. Furthermore, effective diffusion is virtually equal to the local diffusion in diffusion dominated problems.

Keywords: seawater intrusion modeling, Henry Problem, dispersive Henry Problem, heterogeneity, effective dispersion, anisotropy

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Introduction

Seawater intrusion into coastal aquifers exemplifies natural stable density stratification with denser saltwater encroaching below freshwater. An abstraction of the problem of saltwater intrusion in a vertical cross-section perpendicular to the coastline was introduced by Henry (1964). The conceptual model is that of a confined aquifer with homogeneous isotropic hydraulic conductivity, and with the following boundary conditions (BC): no-flow along the top and bottom boundary and specified groundwater seepage of 6.6e-5 m/s with salt mass fraction of zero (freshwater) along the vertical inland boundary and a hydrostatic pressure distribution of saltwater along the vertical seaside boundary. The original Henry Problem considers advection and diffusion (no dispersion). The parameters originally used are presented in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>0.35</td>
<td>Porosity</td>
</tr>
<tr>
<td>$D_m$</td>
<td>1.88571E-05 m²/s</td>
<td>Effective diffusion coefficient</td>
</tr>
<tr>
<td>$K$</td>
<td>1.0e-2 m/s</td>
<td>Hydraulic conductivity</td>
</tr>
<tr>
<td>$Q$</td>
<td>6.6e-5 m²/s</td>
<td>Freshwater total inflow at inland boundary</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>0 kg/kg</td>
<td>Mass fraction of freshwater</td>
</tr>
<tr>
<td>$\omega_s$</td>
<td>0.0357 kg/kg</td>
<td>Mass fraction of seawater</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>1000 kg/m³</td>
<td>Freshwater density</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>1024.99 kg/m³</td>
<td>Seawater density</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.001 kg/m·s</td>
<td>Fluid viscosity (same for both fluids)</td>
</tr>
</tbody>
</table>

This configuration leads to a characteristic, stationary saltwater intrusion wedge, penetrating landward on the aquifer bottom. Henry (1964) provided a semi-analytical solution for this problem configuration. His solution was revised and improved by Borisov et al. (1996) and Segol (1994). The analytical solution in both studies was given as an infinite-series solution. The two main drawbacks of this solution are: (1) it requires a constant, effective diffusion term, and (2) the boundary condition for concentration at the seaside border is of Dirichlet type. The original boundary condition of the Henry Problem at the seaside border was subsequently modified by e.g., Segol et al. (1975) and Frind (1982) to represent a more realistic boundary with freshwater discharge. This modified boundary condition consists of specified pressure along the seaside boundary (Dirichlet type for flow) with solute concentration equal to seawater concentration for inflowing portions and resident concentration for the outflowing boundary’s portion. This transport boundary condition is implemented by specifying the mass flow as shown in equation (1).

$$D \nabla c - qc^* = qc^* \begin{cases} \text{if } q < 0 & \text{then } c^* = c \\ \text{if } q > 0 & \text{then } c^* = c \end{cases},$$ (1)

where $D$ is the hydrodynamic dispersion tensor, $q$ the Darcy’s flux, and $c^*$ is the boundary flux concentration, which is equal to resident concentration, $c$, when aquifer water flows towards the sea and seawater concentration, $c_s$, otherwise. In any case, as shown by the numerical calculations, the choice of BC has a rather small impact on the overall location of the concentration isolines. The boundary conditions selected in our work are those of Frind (1982), as shown in Figure 1.
The Henry Problem has become a classic test case for variable-density flow codes. A good review of the Henry Problem history is described in Segol (1994). However, the influence of the boundary conditions in this type problem suggests that it is not the best test for variable-density codes (Simpson and Clement, 2003). The results of the different numerical solutions show no significant discrepancies for grid Peclet numbers below 1 (Oswald, 1999). References in the literature indicated that this test case is not representative for the numerical treatment of narrow, more realistic, mixing layers at the interface between saltwater and freshwater (Voss and Souza, 1987). Several studies have accounted for variable dispersion to simulate seawater intrusion in this benchmark problem (Frind, 1982; Huyakorn et al., 1987). In this contribution we consider three cases of a modified Henry Problem with respect to the treatment of diffusion/dispersion. A comparison is made between the purely diffusive traditional Henry Problem, a fully dispersive case (no molecular diffusion) and a mixed (diffusive/dispersive) case, also considered in a previous work (Schwarz, 1999).

Heterogeneity in hydraulic properties, especially hydraulic conductivity, is well known to significantly affect groundwater flow and solute transport. In variable-density flow systems, heterogeneity can perturb flow over many length scales, ranging from slight differences in pore geometry to larger heterogeneities at the regional scale. Yet, few studies have focused on evaluating the impact of heterogeneity on variable density flow. Mc Kribbin and O’Sullivan (1980) and Mc Kribbin and Tyvand (1982, 1983) investigated the effect that multiple hydraulic conductivity layers had on thermal convection. Schincariol and Schwartz (1990) carried out an experimental investigation of variable–density groundwater flow in homogeneous, layered, and lenticular media. They found that dense water tends to accumulate along bedding interfaces. Schincariol et al. (1997) numerically reproduced the experimental results of Schincariol and Schwartz (1990).

Schincariol et al. (1997) carried out numerical simulations incorporating the effects of heterogeneity in the hydraulic conductivity field and found that the statistical characteristics of the permeability field (mean, variance, and correlation length) played a critical role in the onset and subsequent growth or decay of instabilities. In the study of Schincariol et al. (1997) random permeability fields were used for the first time in connection with gravitational instabilities. Schincariol (1998) studied natural perturbation initiation by local-scale heterogeneities and concluded that homogeneous field criteria will not be applicable. Simmons et al. (2001) considered different types of heterogeneity and stated that the presence of instabilities was intimately related to the structure and variance of the permeability field. Ordered heterogeneities with vertically oriented high-permeability regions tend to enhance growth conditions while horizontal elongated structures (stochastic distribution) tend to dissipate free convection through dispersive mixing.
All of the described studies focused on instabilities caused by dense fluids overlying lighter ones and not on saltwater intrusion processes. Few investigations have been devoted to the study of the heterogeneity in coastal aquifers. Schwarz (1999) studied the effect of heterogeneity for some of the most typical benchmarks problems for density-dependent flow, including the Henry Problem for saltwater intrusion.

The objective of this work is to assess the effect of heterogeneity in the steady state position of the saltwater intrusion wedge and to analyze whether a homogeneous equivalent approximation can predict the interface penetration and the width of the mixing zone.

**Methodology**

The approach taken in this work is to numerically study the effect of heterogeneity in seawater intrusion problems. The SUTRA code (Voss and Provost, 2002) was used for the simulations of the Henry Problem. The grid is regular, with 256 x 128 elements. This grid discretization was found fine enough to ensure the convergence of the solution and allows a sufficient resolution of the random permeability fields used for the heterogeneous simulations. Benson et al. (1998) studied numerical dispersion in this type of problem (diffusive and dispersive form) using SUTRA and found out that the solution was stable for grid spacing smaller than 4 cm. They concluded that, in highly variable velocity fields, Eulerian models display less error than Lagrangian models, because of the Eulerian uniform first-order truncation error. The boundary conditions used have been discussed above and are shown together with the model domain in Figure 1.

We consider two distinct problems, depending on whether the medium is treated as homogeneous or heterogeneous.

**Homogeneous anisotropic Henry Problem**

The Henry Problem solution depends on three dimensionless numbers: \( a, b \) and \( \xi \):

\[
a = \frac{Q}{K (\rho_s - \rho_o)} \frac{\rho_o}{\rho_s} 
\]

\[
b = \frac{D_s \phi}{Q} 
\]

\[
\xi = \frac{l}{d} 
\]

where \( l \) is the length of the aquifer section and \( d \) is the aquifer thickness.

The semianalytical solution for this problem was evaluated for the following values of these parameters: \( a = 0.263, b = 0.1 \) and \( \xi = 2 \).

The geometry and the boundary conditions of this problem are considered to represent seawater intrusion process realistically; however the produced mixing zone width is overestimated according to most real data.
obtained in electrical conductivity profiles in boreholes. This is due to the high value of the diffusion coefficient used. Another unrealistic aspect is that mixing does not take into account a velocity dependent dispersion. A modified Henry Problem is proposed for this work. The two modifications included are, first, the hydraulic conductivity is anisotropic, and second, we incorporate velocity dependent dispersion. Figure 2 shows the isoconcentration lines for a dispersive anisotropic Henry Problem. The shape and width of the transition zone for the variable dispersion case is considered more realistic and to better represent real data of vertical salinity distribution.

It is necessary to introduce two new dimensionless numbers that define this modified problem together with $a$ and $\xi$. The traditional will Henry Problem constitute a particular case.

The first new number is the anisotropy ratio ($r_h$) and it is defined as:

$$r_h = \frac{k_x}{k_h};$$

where $k_h$ is the horizontal permeability and $k_v$ is the vertical permeability. The permeability values considered in the simulation for the homogeneous cases are presented in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_h$</td>
<td>1.2550700D-09 m²</td>
</tr>
<tr>
<td>$k_v$</td>
<td>8.3805601D-10 m²</td>
</tr>
<tr>
<td>$r_h$</td>
<td>0.66</td>
</tr>
</tbody>
</table>

The $b$ parameter was modified to take into account not only constant diffusion but also velocity dependent dispersion. Different values of the $b$ parameter were studied:

$b_L$: Considering only the local longitudinal dispersivity coefficient ($\alpha_L$).

$$b_L = \frac{D_{mL} + \alpha_L \cdot q}{Q}$$

Figure 2. Concentration isolines for a homogeneous anisotropic dispersive Henry Problem simulated with SUTRA code. Notice the more realistic shape of the transition zone. The anisotropy ratio ($r_h=K_v/K_h$) for this simulation is 0.66.
\( b_T \): Considering only the transversal dispersivity coefficient (\( \alpha_T \))

\[
b_T = \frac{D_m \phi + \alpha_T q}{Q}
\]  

\( b_{geo} \): Considering the geometric mean of those two coefficients (\( \alpha_{geo} \)).

\[
b_{geo} = \frac{D_m \phi + \alpha_{geo} q}{Q} ; \text{ where } \alpha_{geo} = \sqrt{\alpha_T \alpha_L}
\]

The local longitudinal dispersivity coefficient (\( \alpha_L \)) used for the dispersive case is chosen so that it is equal to Henry’s original \( b \) value and the transversal dispersivity coefficient (\( \alpha_T \)) is a tenth of the longitudinal one. That is,

\[
\alpha_L = \frac{D_m \phi}{Q}
\]

\[
\alpha_T = \frac{\alpha_L}{10}
\]

The reference cases used in this study and their values of the dispersivity coefficients are listed in Table 3.

<table>
<thead>
<tr>
<th>CASES</th>
<th>( D_m ) (m²/s)</th>
<th>( \alpha_L ) (m)</th>
<th>( \alpha_T ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diffusive</td>
<td>1.88571E-05</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Mixed</td>
<td>9.42855E-06</td>
<td>0.05</td>
<td>0.005</td>
</tr>
<tr>
<td>Dispersive</td>
<td>0</td>
<td>0.1</td>
<td>0.01</td>
</tr>
</tbody>
</table>

The parameter \( b_{geo} \) correlates better with the obtained results for a set of different simulations with different values of \( \alpha_L \) and/or \( \alpha_T \). Therefore the results of this study are always compared to this \( b_{geo} \) parameter.

For the different scenarios considered we analyze our results in terms of:

- \( L_{toe} \): Penetration of the seawater intrusion wedge measured as the distance between the seaside boundary and the point where the 50% mixing line touches the aquifer bottom (see Figure 3).
- WMZ (Width of the Mixing Zone): It is measured as the mean width between the 25% and 75% mixing lines. In order to avoid problems due to boundary effects this width was considered only between the 0.2·\( L_{toe} \) and 0.8·\( L_{toe} \) (see Figure 3).
- SMF (Salt Mass Flux): Mass of salt that comes into the system through the seaside boundary (kg of salt/s). This parameter is critical for the salt mass exchange across the saltwater/freshwater interface.

A sensitivity analysis was performed for the two new parameters (\( r_k \) and \( b_{geo} \)) in a homogeneous medium. In Figure 4, the seawater wedge penetration (\( L_{toe} \)), the width of the mixing zone and the salt mass flux dependence on \( b_{geo} \) is shown for the three considered cases (fully diffusive, fully dispersive and mixed). The higher \( b_{geo} \), the shorter the seawater penetration, and the thicker the mixing zone gets. The incoming salt...
mass flux also increases with the $b_{geo}$ value. Figure 5 shows the results of the dependence on the anisotropy index ($r_k$). Little effect is observed both in the penetration and in the width of the transition zone. However, as it may be expected, the effect in the incoming mass flux is relevant: the higher the vertical permeability, the higher the amount of salt mass flux.

**Figure 3.** Sketch showing some of the values used for comparison between the different scenarios.

**Figure 4.** Results of the three studied anisotropic cases and their relation to $b_{geo}$

**Figure 5.** Results of the three studied anisotropic cases and their relation to the anisotropy ($r_k$)
The amount of salt mass flux entering the system through the seaside boundary is controlled by the mixing phenomena in the transition zone, i.e., by diffusion or dispersion. In the dispersive dominant cases, flow in the mixing zone is almost parallel to the isoconcentration lines; that means that mixing is controlled predominantly by the transverse dispersivity. Diffusion/dispersion also controls the width of the mixing zone; therefore, a relation between the saltwater mass flux and the width of the mixing zone is expected. This relation is plotted in Figure 6a for different values of $b_{geo}$. An almost linear relation is shown, but two different slopes can be distinguished, one for the diffusive dominant cases and a second one for the dispersive case. In Figure 6b, the same result is plotted but for simulations with different anisotropy ratio. There is no variation in the width of the mixing zone for the dispersive case, although for a higher $r_k$, the vertical permeability is higher and thus, salt inflow also increases. The relation for the diffusion dominating cases for different $r_k$ values is inverse. There is an increase in the seawater mass flux but a slight reduction in the width of the mixing zone for high values of $r_k$.

Heterogeneous anisotropic Henry Problem

To evaluate the effect of heterogeneity and the equivalent diffusion/dispersion coefficients in an equivalent homogeneous medium approach, 60 random permeability fields were generated. The heterogeneous fields were generated using the GCOSIM3D code (Gomez-Hernandez and Journel, 1993). The hydraulic conductivity distribution is assumed to follow a log-normal probability density function. The geostatistical model used is spherical with anisotropic correlation lengths of 0.15 m and 0.045 m in horizontal and vertical directions, respectively, with a mean value equal to the logarithm of Henry’s hydraulic conductivity and a variance of 1. The anisotropic Henry Problems described in the previous sections were simulated for each of the heterogeneous realizations and for the three dispersion cases: diffusive, mixed and dispersive. The results of those simulations were compared to their corresponding homogeneous solution. The homogeneous effective media to compare with were built with the following effective parameters:
the effective permeability in x and y directions is the mean value of the horizontal and vertical permeability of the set of 60 heterogeneous realizations.

the values of the diffusivity/dispersivity coefficients were set equal to the local coefficients used in the heterogeneous simulations.

Results

Some examples of the solutions obtained for the concentration distribution in heterogeneous dispersive cases are shown in Figure 7. Some patterns that appear in this solution are general for all the heterogeneous results:

– As expected, the slope of the interface is lower in the high permeability zones while the slope increases in the low permeability zones.

– Isolines tend to accumulate in high permeability zones (white) while they tend to spread out in low permeability zones. For this reason, isolines seem to accommodate under high permeability zones.

Figure 7. Solution of the concentration distribution for four heterogeneous dispersive realizations. Black lines show the 10-20-30-…-90% mixing lines.

$L_{toe}$ and width of the mixing zone (WMZ)

In the following paragraphs, we present the results on the penetration of the saltwater wedge and on the width of the mixing zone for the heterogeneous simulations, their comparison with the homogeneous cases with local values of dispersion, and how the agreement between both results could be improved.
The distribution of the results of the heterogeneous simulations is shown in Figure 8a ($L_{\text{top}}$) and Figure 8b (width of mixing zone) in box plots for each of the three considered cases. On the left of these box plots the results for the homogeneous cases are plotted. The horizontal dashed lines are drawn for the comparison between results. A description of the results for each of the considered dispersion cases follows.

**Diffusive case**  
The solution obtained with the homogeneous effective medium represents quite well the mean of the heterogeneous results distributions for the purely diffusive case, both in terms of penetration and width of the mixing zone. Therefore, the local value of the diffusion coefficient used in the homogeneous case seems to be the appropriate parameter to represent the mean heterogeneous behaviour.

![Boxplot diagrams](image)

*Figure 8.* Boxplot diagrams of the distribution of the $L_{\text{top}}$ (top) and width of the mixing zone (bottom) for the heterogeneous simulations and their corresponding plot of the homogeneous medium results. The horizontal lines are drawn for comparison purposes.
Mixed case
In the mixed case the WMZ is well represented by the homogeneous medium, while the toe penetration is overestimated. To reduce the toe penetration for the effective case, a slight increase in the diffusion/dispersion coefficient \( b_{geo} \) would be needed as was shown in the sensitivity analysis for the homogeneous case. But at the same time, this would lead to an increase in the width of the mixing zone that would worsen the WMZ comparability. Therefore, it is not appropriate to increase the \( b_{geo} \) value of the homogeneous effective system for the mixed case.

Dispersive case
The results for the purely dispersive homogeneous case diverge more from the mean of the heterogeneous results both in terms of \( L_{toe} \) and WMZ. Both fits would be improved by increasing the value of the dispersivity coefficient, that is, the value of the \( b_{geo} \) parameter.

Seawater mass flux
The relation between the SMF and the WMF for the heterogeneous cases is represented in Figure 9. Seawater mass flux is in, virtually, all cases, underestimated in the homogeneous simulations. The salt inflow is, in most of the heterogeneous realizations, remarkably higher than the salt mass flux in the homogeneous case. Moreover, the realizations that present a higher amount of salt through the seaside boundary are the same for the three considered cases (diffusive, mixed and dispersive). Therefore, the SMF is controlled more by the permeability field than by the diffusion/dispersion mechanism.

![Figure 9](image.png)

Figure 9. Relationship between the SMF and the WMF for the heterogeneous simulations and the homogeneous results.

Figure 9 also shows that the WMZ has a slightly inverse correlation with the SMF for the heterogeneous diffusive cases. This effect is also observed in the simulations for the mixed case. However, for the dispersive cases, there is not such a relationship between the amount of salt that comes into the aquifer, that is highly variable, and the WMZ that stays more or less constant. These relations were also observed for the
anisotropic homogeneous case as the vertical permeability or $r_k$ value is increased (Figure 6b). Therefore, the increase in the SMF is due to locally higher vertical permeability in different heterogeneous realizations near the seaside boundary.

Discussion

When comparing the results of the heterogeneous simulations with those of a homogeneous anisotropic medium with the same local diffusion/dispersion coefficient, the results of the penetration width of the mixing zone compare favorably. Only for purely dispersive cases, a better fit would be accomplished with a slight increase in the equivalent dispersion coefficient. The seawater mass flux is however underestimated in the homogeneous effective cases with local dispersion coefficients. The higher amount of salt that comes into the system in the heterogeneous media is related to the local increase of the vertical permeability near the seaside boundary.

Acknowledgements

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References


