Dispersive Behavior of the Mixing Zone between a Shallow Freshwater Lens and Upward Seeping Saline Groundwater

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ABSTRACT
This study focuses on the mixing zone between thin, shallow freshwater lenses and underlying, upward seeping saline groundwater, under homogeneous isotropic conditions. The role of longitudinal and transverse dispersivities is analyzed at different phases of lens formation, increasing insight into the mixing processes. Spatial moments are used to characterize the mixing zone in time and spatially.

INTRODUCTION
Salt water intrusion and upward seepage of saline groundwater is a widespread problem in low-lying areas of coastal zones that are important for agriculture or ecologically, such as in the North and West regions of the Netherlands, Southwest Florida, and many other deltaic areas such as the Camargue in France and the Nile delta in Egypt. Where soil surface is situated below sea level, the pressure of this sea level moves saline groundwater upward into superficial water networks. On the other hand, infiltrating rainwater forms fresh water lenses. Phreatic rise is limited by the surface elevation, which in its turn limits the maximum possible depth of the fresh water lens (Ghijben Herzberg).

Changing climate and sea level rise may change the delicate balance between fresh and saline water flows. Climate change may increase drought due to reduced differences between rainfall and evapotranspiration, an increase of dry periods, and a decreased availability of irrigation water from rivers in such periods (Beersma *et al.*, 2004). Sea level rise also directly increases the driving force of the seepage flux of saline groundwater up to tens of kilometers from the coast (Oude Essink, 2001). Change in this balance between fresh and saline water fluxes may influence the availability of fresh water in the root zone. Hence, the thickness of both the fresh water lens and of the mixing zone is important for crop yield and natural vegetation diversity.

So far, studies have mainly focused on large-scale seawater intrusion. Here the transition zone is assumed negligible, as it is commonly relatively thin compared to the lens thickness (Oude Essink, 2001). This assumption is valid for cases with either a small dispersion to convection ratio or a large seepage factor (flux to conductivity ratio) (Sakr, 1999). For small (single field) scales, neither are the case.

Our primary aim is to quantify the shape of fresh water lenses, taking into account the transition zone between fresh and saline water, which we define as the second central moment of concentration change with depth. The set of dimensionless parameters that we analyzed is valid on all scales. However, we expect our results to be most interesting on the field scale, defined as the distance between two parallel ditches or other forms of drainage such as pipe drainage. This scale is most relevant for assessing damage to crops or natural vegetation due to saline water entering the root zone.

THEORY

Physical description of the system

The situation we are interested in is a vertical cross section through a long, narrow field between 2 ditches (see Figure 1). The width of a half field, which is sufficient for symmetry-reasons, is set at 25 meters. The ground water level is high, generally around one meter below surface, which is a common situation in Dutch polders and similar areas. Dispersivities are estimated at 0.25 m (longitudinal) and 0.025 m (transverse). In the given system, the development of rainwater lenses on saline groundwater is influenced by different factors, such as water fluxes and properties, soil properties and geometry. The upper boundary is a flux boundary, where net precipitation causes infiltrating fresh water. The side boundaries are no-flow boundaries, both are hydrological divides. The lower boundary is a flux boundary where saline groundwater seeps upward into the system, caused by the position of the field below sea level.

Numerical modeling approach

We used SUTRA for our numerical modeling (Voss and Provost, 2003*)*, which is a physically based numerical code, that is able to model density dependent saturated and unsaturated flow in two or three dimensions. It solves the classical equations for flow and transport. The model has been verified for density dependent flow situations. We chose our time steps and mesh density such that numerical dispersion remained smaller then mechanical dispersion.

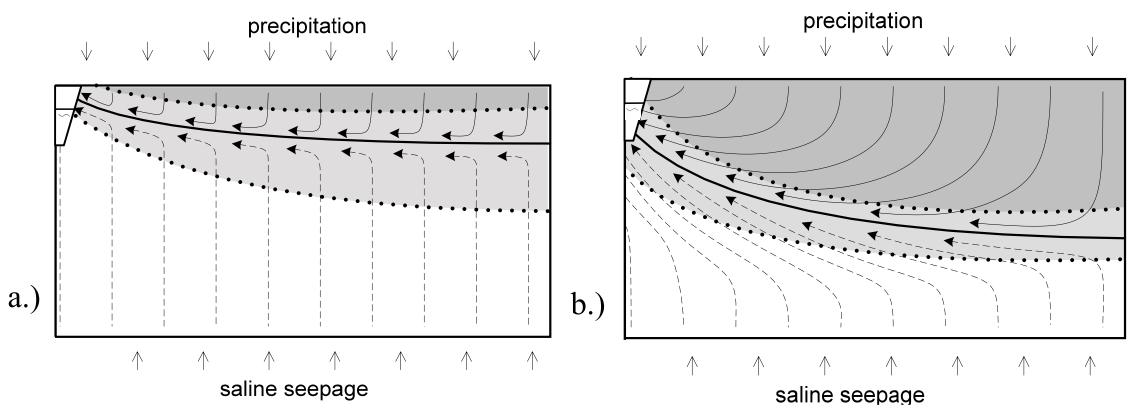

Figure 1. Flow lines of infiltrating fresh water (grey) and upward seeping saline water (white), from the ditch to the middle of a field. a.) during initial lens formation and b.) at steady state (right). Solid lines: imaginary sharp interface. Dotted lines: observed mixing zone

RESULTS

Dispersive behavior

Together with molecular diffusion, which is assumed to be of minor importance, dispersivity is the main parameter determining the width and development of the mixing zone. Therefore, we investigated the effect of dispersion on the development of the mixing zone. To quantify these processes, we defined the interface as the line that connects the vertical first moments of concentration change. From the numerical results, we determined the velocity components in parallel and perpendicular directions with respect to the interface.

Using the dispersion-velocity relations as formulated by (Bear, 1979) we determined the relative contributions of longitudinal and transverse dispersion to the total dispersion flux perpendicular to the interface. These relative contributions are given by

$$\frac{\alpha_l v_t^2}{\alpha_l v_t^2 + \alpha_t v_l^2} \quad \text{(longitudinal contribution)} \qquad (1)$$

$$\frac{\alpha_t v_l^2}{\alpha_l v_t^2 + \alpha_t v_l^2} \quad \text{(transverse contribution)} \qquad (2)$$

Here v_l , v_t , α_l and α_t are the longitudinal and transverse components of the velocity and dispersivity, respectively. The contribution of longitudinal and transverse dispersion to mixing was determined for a fresh water lens that develops in an initially saline domain. Longitudinal dispersivity was assumed to be an order of magnitude larger than transverse dispersivity. The relative contributions of longitudinal and transverse dispersion are shown while the lens develops, and at steady state conditions in Figure 2.

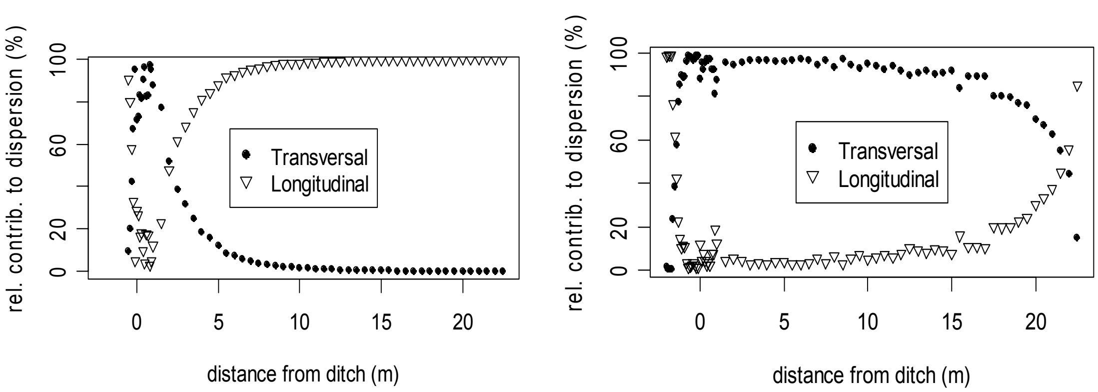

Figure 2: dispersion during lens development (after 1 year; left) and at steady state (right).

Close to the hydrological divide in the middle of the field, dispersivity is highly influenced by boundary effects. Transverse dispersion is always dominant near the ditch boundary, due to the high, velocities, which are nearly parallel to the mixing zone. The curvature in the mixing zone in this area complicates determination of parallel and perpendicular velocity in this zone: in equations (1) and (2), longitudinal ad transverse velocity components are needed. These components (respectively parallel and perpendicular to the interface) can only be calculated well if the interface is almost horizontal, due to the rectangular grid. Near the ditch the interface is significantly tilted and the different velocity components are not well estimated.

Disregarding the boundary regions, near the ditch and near the water divide, we observed that longitudinal dispersion is significant during lens development (Figure 2), and increases towards the centre of the field. When the lens has reached its steady state shape, the influence of longitudinal dispersion disappears almost entirely (Figure 2). The mixing zone now shifted towards an equilibrium situation, more or less parallel to the flow direction (Figure 1). The dominance of longitudinal dispersion during lens development leads to a significantly larger mixing zone at this stage, due to the larger value of longitudinal dispersivity.

DISCUSSION AND CONCLUSIONS

The phase of lens formation determines the importance of longitudinal and transverse dispersion, respectively. This observation can be understood from the different flow directions and velocities of the precipitation and seepage fluxes in different phases.

Since the dynamics of lens formation for small and large scale situations are characterized by different characteristic times, also the importance of longitudinal and transverse dispersion may be expected to be scale dependent.

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