Analytical Method for Preliminary Management of Pumping and Injection in Coastal Areas

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ABSTRACT
In this work two algebraic equations are developed to estimate the maximum pumping rates and the minimum injection rates, respectively, subject to a pre-specified saltwater intrusion limit and a set of simplifying assumptions. Strack's single-potential solution is used to derive these equations. In general, the maximum pumping rate increases as more additional intrusion is allowed. However, critical points limit both the maximum pumping rate and the allowed saltwater intrusion limit. For injection rates there is no such limit. Design curves are developed from the equations.The equations can be presented as design curves for ease of use.

INTRODUCTION
Ground water development in coastal areas induces saltwater intrusion. If not developed carefully, saltwater may reach the pumping well. This type of problem is a typical optimization problem: maximizing the withdrawal while limiting the saltwater intrusion to a specified limit. Another type of optimization problem exists: In cases where aquifer is already saltwater contaminated, freshwater may be injected to push the saltwater toward the sea to reclaim the aquifer. In this case, minimization of freshwater injection while limiting the saltwater intrusion to a desired location is another typical optimization problem.

For general problems simulation and optimization techniques (Park and Hong, 2006) are needed to determine maximum pumping or minimum injection rates. However these techniques require detailed investigation and require intensive computing capabilities which may not be available for a preliminary assessment.

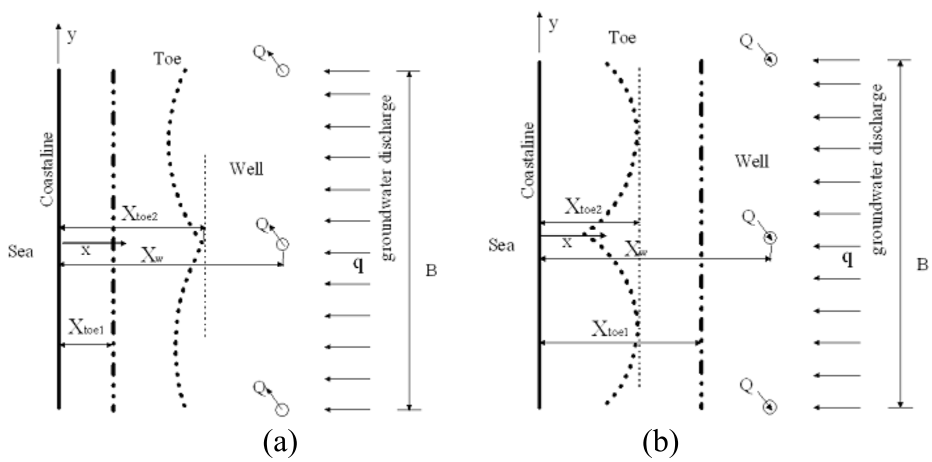


Figure 1. Plan views of coastal area for (a) pumping and (b) injecting freshwater

Prior to pumping or injection, the toe of a saltwater wedge, distance x_{toe1} away from the coastline, is aligned parallel to the coastline when the coastal groundwater discharge is uniform. When groundwater is pumped or freshwater is injected, the toe shall react. A preliminary management goal would be to estimate for groundwater development projects the maximum pumping rates or for aquifer restoration projects the minimum injection rates. Both types of projects are subject to a desired post-development maximum intrusion location (x_{toe2}, Figure 1).

When certain conditions are met, the maximum or injection rates can be determined directly without resorting to an optimization technique. In this work algebraic equations are developed and presented in the forms of design curves for quick assessment on groundwater development in planning stage with available hydrogeologic data.

ANALYTICAL METHOD

As metioned above, a new algebraic equaiton to estimate the maximum pumping rate and the minimum injection rate can be derived from Strack's single-potential solution for sharp interface moder. It is developed by Cheng and other researchers to determine the respose of the toe position to groundwater development via multiple wells. The solution shows the relationship between pumping or injection rate Q_i, the post-development toe coordinate (x, y), and well position coordinates (x_i, y_i)as follow:

$$\phi_{toe} = \frac{\lambda}{2} l^2 = \frac{q}{K} x + \sum_{i=1}^{N_w} \frac{Q_i}{4\pi K} \ln\left[\frac{(x-x_i)^2 + (y-y_i)^2}{(x+x_i)^2 + (y-y_i)^2}\right] \tag{1}$$

where λ is set as $s(s-1)$and l is set as d for the unconfined aquifer, $\lambda = s - 1$ and $l = D$ for the confined aquifer. Herein D is the confined aquifer thickness, d is the elevation of mean sea level above the datum, s is the density ratio of the saltwater and freshwater(ρ_s / ρ_f). The groundwater discharge is expressed as q, and the N_w is the total number of wells. When there is no pumping or injection, i.e. Q_i=0, the pre-development toe position (x_{toe1}) can obtained from equation (1).

For the sake of applying the equation, the odd number of wells are assumed and distribution of the wells is symmetrical and parallel the coastaline, which is limited in a given length B. Such setting can guanarantee a well always placed along y=0 axis, thus the well coordinates can be express as $x_i = x_w$ and $y_i = -nb, \cdots, -b, 0, b, \cdots nb$, where $n = (N_w - 1)/2$.

Then assume that all the wells seperate from each other with the same distance, and the Q_i,has the same value, that is, each of the wells share the same pumping or injection rate. Additional assumptions can be employed for applying equation (1) for priliminary management. We limit our research into a B length homogeneous isotropic coastal area, distribute odd number wells symmetrically parallel the coastline, and assume all wells sharing same well space b, pumping rate or injection rate Q . In terms of the previous assumption with some defined dimensionless variables, like $\eta = x_{toe} / x_w; \quad \mu = y_{toe} / x_w; \quad \eta_1 = x_{toe1} / x_w; \quad \beta = b / x_w; \quad \xi = Q / \pi K l^2$, the equation can be simplified to

$$\xi = 2\lambda \left(\frac{\eta - \eta_1}{\eta_1}\right) \left\{ \sum_{i=-n}^{n} \ln\left[\frac{(\eta-1)^2 + (\mu - n\beta)^2}{(\eta+1)^2 + (\mu - n\beta)^2}\right] \right\}^{-1} \tag{2}$$

The dimensionless equation of ξ is a function of dimensionless pre-development toe position η_1, dimensionless post development toe coordinates (η, μ) and well space β. From above equation, the variale ξ primarily vary with the post-development toe location under given aquifer with know well space. Due to the specified limit on saltwater intrusion (η_2, defined as x_{toe2}/x_w), there is maximum pumping rate or minimum injection rate, which will be obtained while maximum post-development toe (η) equal to η_2.

MAXIMUM PUMPING RATE

As shown in Figure 1(a), the post development toes move symmetrically because of the symmetrical distribution of the wells, hence the maximum saltwater intrusion will also happen along the y= 0 axis. Substitute dimensionless maximum intrusion coordinate (η_2, 0)into equation (2) will yield

$$\xi_{\max} = 2\lambda \frac{\eta_2 - \eta_1}{\eta_1} \left[\sum_{i=-n}^{n} \ln \frac{(1-\eta_2)^2 + (i\beta)^2}{(1+\eta_2)^2 + (i\beta)^2} \right]^{-1} \tag{3}$$

From above equation, it seems that the dimensionless pumping rate ξ_{max} is proportional to additional intrusion length, and by allowing more intrusion more ground water can be developed. However, as the intrusion approach a special toe called critical point x_c, an unstable situation will happen, at which the slightest increase of pumping rate will cause the saltwater intrusion into the well. It can be defined by requiring the stagnation point correspond to the toe position as

$$\eta_1 = \eta_c + \frac{1-\eta_c^{\ 2}}{4} \sum_{i=-n}^{n} \ln \left[\frac{(\eta_c - 1)^2 + (i\beta)^2}{(\eta_c + 1)^2 + (i\beta)^2} \right] \tag{4}$$

where η_c is dimensionless value of x_c by x_w

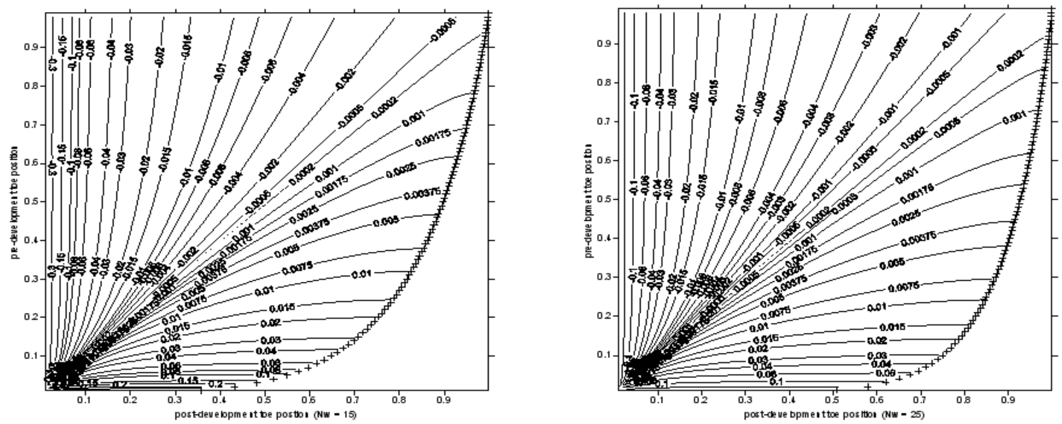
MINIMUM INJECTION RATE

As shown in Figure 1(b), The effects of injection would be more pronounced along the symmetry line than near the edge of the well, thus the maximum intrusion would occur somewhere between the two marginal wells. The location of the maximum intrusion herein is pointed as the middle point and has been proved can be safely used to derive the minimum injection rate equation. Therefore, the dimensionless maximum intrusion coordinate becomes (η_2, (n-0.5)β) which yield

$$\xi_{\min} = 2\lambda \frac{\eta_2 - \eta_1}{\eta_1} \left(\sum_{i=-n}^{n} \ln \left[\frac{(1-\eta_2)^2 + [(n-0.5)\beta + i\beta]^2}{(1+\eta_2)^2 + [(n-0.5)\beta + i\beta]^2} \right] \right)^{-1} \tag{5}$$

DESIGN CURVES

The two equations can be combined into design curves for both pumping and injection cases with a given number of wells. Figure 2 depicts the design curves for 15- and 25-well cases. Values of the contour lines in Figure 2 represent the uniform pumping rate and injection rate from a single well for a combination of a predevelopment of toe position (η_1) and a desired limit of saltwater intrusion (η_2). The curve marked by ‘+’ represents the critical points, and the blank areas at the right bottom is invalid for the sake of avoiding saltwater contaminated the well.



(a) (b)

Figure 2. Design curves for operation rates of individual well 15 (a) and 25 (b) wells.

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