Benchmarks for Two- And Three-Dimensional Variable-Density Ground-Water Flow Simulators: Analytical Expressions for Unstable Convection

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EXTENDED ABSTRACT

Approximate stability criteria for unstable steady-state density configurations, determined analytically, may be used as benchmarks for both two-dimensional (2D) and three-dimensional (3D) numerical simulators of variable-density ground-water flow. One of the more difficult tasks of variable-density simulators is to correctly represent the physics of unstable density configurations; thus, these benchmarks may be among the more stringent tests of such computer codes.

In 2D, the benchmarks consist of approximate analytical stability criteria for steady unstable convection in a finite 2D box with top and bottom boundary conditions that maintain an unstable density configuration (dense fluid above less-dense fluid). These are closely related to the classic parallel plate analyses of Horton and Rogers (1945) and Lapwood (1948).

2D models may be tested by:

1- matching numerical results with analytical results for convective wavelength and critical Rayleigh number (Ra_c) for very wide horizontal boxes (wavelength results are sensitive to initial seeding, as described below),

2- matching Ra_c as a function of horizontal box geometrical aspect ratio with an analytical expression, and,

3- matching Ra_c as a function of box inclination with an analytical expression.

These 2D benchmarks were proposed by Weatherill et al. (2004).

In 3D, proposed benchmarks consist of approximate analytical stability criteria for steady unstable convection in a finite 3D box. These benchmarks are analogous to those for 2D, but refer to the regimes of convection that occur in 3D: polyhedral cells (occur for low angles when $Ra > Ra_c$), unicellular convection (occurs for all angles greater than zero when $Ra < Ra_c$), and longitudinal coils (occur for high angles when $Ra > Ra_c$). The 3D stability criteria are corrected from classic analyses of 3D convection in porous media that were reviewed by Caltagirone (1982).

3D models may be tested by:

1- matching numerical results with analytical results for convective wavelength of polyhedral cells and critical Rayleigh number (Ra_c) for horizontal boxes (wavelength results are sensitive to initial seeding, as described below),

2- matching the transition between unicellular and the other convective regimes as a function of box geometrical aspect ratio and box inclination, given by an analytical expression for Ra_c, and,

3- matching simulated transverse wavelength for the longitudinal coil regime.

At low angles of inclination, steady-state solutions are highly sensitive to the initial condition. This imposes two constraints on initial conditions, if simulation results are to be matched with analytical expressions. First, the initial condition must be a low Rayleigh number solution ($Ra < Ra_c$) that provides a quiescent initial state. For unstable energy transport, initial temperature is a vertically-decreasing linear function. For unstable solute transport, the initial concentration, expressed as a mass fraction of solute, is a non-linear function with increasing value along the vertical coordinate. This is a result of the mathematical expression of the diffusion process that dominates for such low Ra solutions. Second, the organization of steady-state convective flow as cells, particularly the convective wavelength, is sensitive to seeding of the initial quiescent state because a family of possible stable solutions exists. Without seeding superimposed on the above-described initial conditions, the simulated steady-state organization may be sensitive to otherwise innocuous numerical irregularities in the solution method used, which provide an unintentional seeding. This was demonstrated for the 2D box by Ataie-Ashtiani and Aghayi (2006).

If benchmarks that rely on seeding (specifically, comparison of simulated and analytical convective wavelength for low-angle boxes) are judged to be overly constrained and thus weak tests, the other proposed benchmarks remain as seeding-independent tests. These include: for 2D, matching the critical Rayleigh number (Ra_c) for any inclination and box aspect ratio, and for 3D, matching the theoretical transition between unicellular and longitudinal coil regimes for any inclination and box aspect ratio, and matching the wavelength of longitudinal coils, which occur irrespective of initial seeding.

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