

## Dimensional parameters of the Henry problem when diffusive effects are negligible

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### ABSTRACT

The parameters that determine the patterns of non-dispersive, anisotropic aquifer of the steady state Henry problem, when diffusion effects are small or negligible are discussed. For this scenery, three dimensionless groups emerge by doing a dimensional analysis for example, two of them are the specific permeability and the shape factor –as defined by Henry– and the other the horizontal and vertical permeability ratio –as defined by other researches. Two different groups of these parameters are used to simulate the problem by network method and to obtain the final patterns.

### INTRODUCTION AND NOMENCLATURE

Despite Henry salt intrusion problem (Henry, 1964) has been highly controversial, it has enjoyed great popularity as a test case to benchmark many density-driven groundwater flow codes (Frind, 1982 and Voss and Souza, 1987, for example). In his paper, Henry deals with the nondimensional form of the governing equations and presents a semianalytical solution for the steady state patterns of iso-chlors and stream function lines. This solution has long been compared with those of other authors. A clear and precise exposition of what Henry benchmark problem has supposed in the history of groundwater flow and solute transport studies can be read in Croucher and O’Sullivan (1995). Henry formulated his problem for the homogeneous, isotropic aquifer, Figure 1.

Assuming the Boussinesq approximation,  $\Delta\rho = \rho_o \beta(\Delta c)$ , and Darcy’s law,  $(\mu/k)\mathbf{q} + \nabla p - (\Delta\rho)\mathbf{g} = 0$ , the governing equations are

$$\nabla \mathbf{q} = 0, \quad (1)$$

$$\frac{\partial c}{\partial t} - \mathbf{v} \cdot \nabla \mathbf{c} = \nabla \cdot (\mathbf{D} \nabla \mathbf{c}) \quad (2)$$

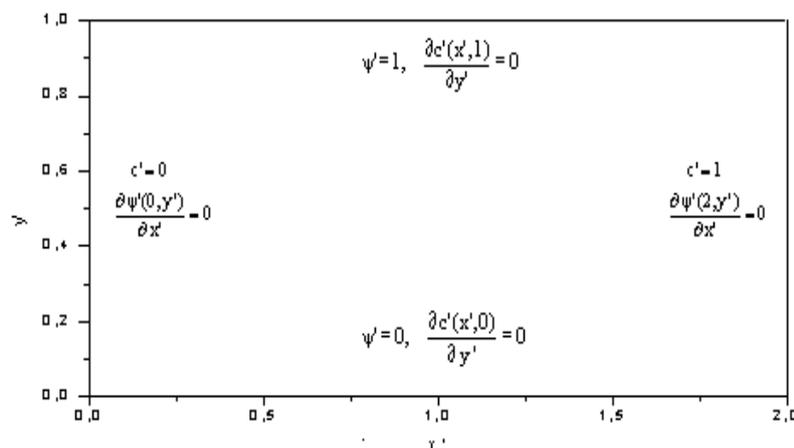


Figure 1. Physical model of the Henry problem

### Nomenclature:

$c, c_s$	concentration of salt in saltwater and seawater	$D$	salt dispersion coefficient ( $m^2/s$ )
$L$	length of the aquifer (m)	$g$	gravitational acceleration (m/s)
$H$	depth of the aquifer (m)	$k$	aquifer permeability
$p$	pressure (Pa)	$q$	specific discharge $=\varepsilon v$ (m/s)
$Q$	volumetric freshwater inflow rate ( $m^2/s$ )	$v$	groundwater velocity (m/s)
$v_{amb}$	regional ambient velocity (m/s)	$x, y, t$	spatial coordinates (m) and time (s)
$\varepsilon$	porosity (dimensionless)	$\mu$	dynamic viscosity of saltwater ( $kg\ m^{-1}s^{-1}$ )
$\rho_s, \rho_o$	saltwater and freshwater densities ( $kg\ m^{-3}$ )		scripts $x$ and $y$ alludes to horizontal and vertical

With the dimensionless variables ( $x'=x/H$ ,  $y'=y/H$ ,  $q'=(H/Q)q$ ,  $c'=\rho'=(\rho-\rho_o)/(\rho_s-\rho_o)=c/c_s$ ) and using the stream function,  $q_x=\partial\psi/\partial y$  and  $q_y=-\partial\psi/\partial x$ , the nondimensionalization of the equations leads to a set of three dimensionless groups:

$$a=(kg\Delta\rho)/(\varepsilon\mu v_{amb}), \quad b=\varepsilon D/Q=D/(v_{amb}H), \quad \xi=L/H$$

For the case of anisotropic aquifer, a new parameter must be added, the permeability ratio  $k_x/k_y$ , being one of these directional permeability assigned to the parameter  $a$ . In her PhD, Abarca (2006) studies a complete scenery that includes dispersive plus diffusive effects, finding four parameters, those of Henry plus the permeability ratio. If, really, both diffusive plus dispersive effects are assumed negligible, parameter  $b$  should be hardly influential, reducing the dimensional groups of this case to three

$$a=(\mu Q/k_y g H)(\rho_s-\rho_o)^{-1}=\varepsilon\mu v_{amb}/(kg\Delta\rho), \quad \xi=L/H, \quad c=k_x/k_y$$

The purpose of this work is to carry out an initial investigation of this case.

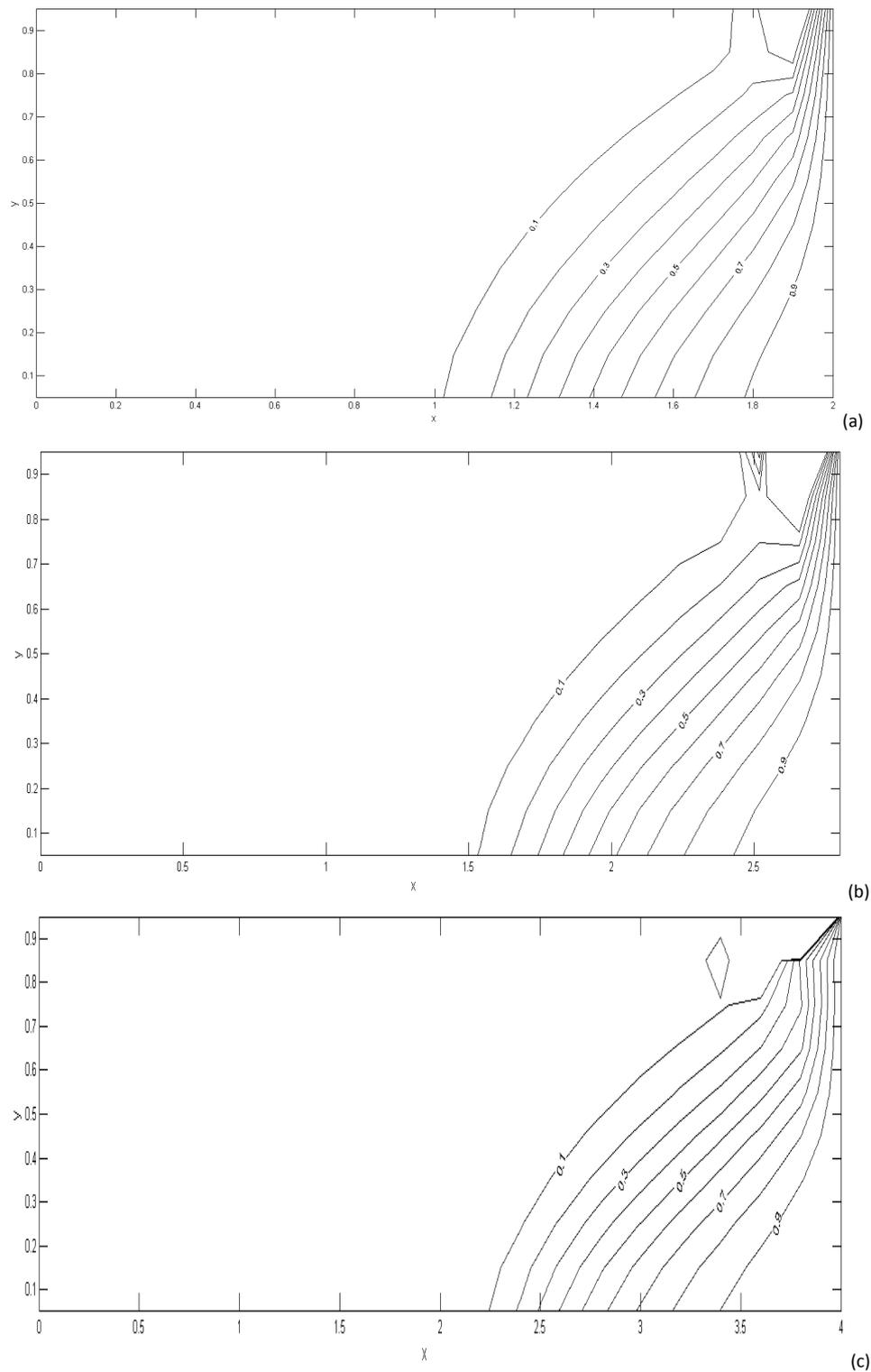
## SIMULATION RESULTS

Table 1 shows the selecting cases to be simulated and the values of the changing parameters. The rest,  $H=1\ m$ ,  $D=1.8857E-5\ m^2/s$ ,  $k_y=1.02e-9\ m^2$ ,  $g=9.81\ m/s^2$ ,  $\Delta\rho=25\ kg/m^3$ ,  $\mu=1E-3\ kg/(ms)$ ,  $v_{amb}=1.88E-4$ ,  $a=\varepsilon\mu v_{amb}(k_y g \Delta\rho)^{-1}=0.263$  and  $\varepsilon=0.35$ , remain constant. Values of  $k_x$ ,  $L$  and  $c$  were chosen suitably in order to obtain similar patterns for the three cases. Simulation was carried out by network method (González-Fernández, 2002). Model used ( $20\times 10$  cells) is detailed in Soto *et al.* (2006).

**Table 1. Values of the parameters to be simulated (SI)**

Case	$k_x$	$L$	$c(k_x/k_y)$
1	1.02e-9	2	1
2	2.04e-9	2.82	2
3	4.08e-9	4	4

Figure 1 shows the isoconcentration patterns of the three cases which have been represented in the same size –i. e., with the same lengths for the depth and width, a kind of, let us say, picture normalization– for a better comparison; local coordinates, however, were maintained independently for each case. As seen, the three patterns are nearly the same for this particular combination of values of  $k_x$  and  $L$ ; concentrations at the same relative locations are very similar. Note, however, that both horizontal permeability and length of the aquifer have changed appreciably:  $k_{x,case\ 3} = 2k_{x,case\ 2} = 4k_{x,case\ 1}$  while  $L_{case\ 3} = (2)^{0.5} L_{case\ 2} = 4L_{case\ 1}$ .



**Figure 1. Isoconcentration lines. (a): case 1, (b): case 2, (c): case 3**

According to these results, it seems that the dimensionless parameters  $\xi=L/H$  and  $c=k_x/k_y$  do not play, separately, and independent roll in the problem but, probably, the new dimensionless group formed by a combination of them for which the value does not change for the three cases. This is  $(k_x/k_y)(H/L)^2$ , a kind of 'discriminated' dimensionless group that joins physical and geometrical parameters that frequently appears, for example, in heat transfer problems (Alhama and Madrid, 2007). If these results are later confirmed by a more refined study, the dimensionless parameters

of the diffusive, dispersive and diffusive plus dispersive Henry problem would also be revised by applying the discrimination concept.

## CONCLUSIONS

The paper of permeability ratio as a parameter in the non-diffusive and non-dispersive Henry problem, with anisotropic permeability, has been discussed in order to investigate if the dimensionless parameters  $\xi=L/H$ , the aspect ratio of the aquifer, and  $c=k_x/k_y$ , the permeability ratio, play an independent role in the steady state patterns of the solution. The influence of parameter  $b=\varepsilon D/Q=D/(v_{amb}H) = 0.1$ , that balances diffusive and regional flow forces, is assumed negligible in the pattern due to its small and constant value. From this initial study, it really seems that  $L/H$  and  $k_x/k_y$  do not play an independent role separately in the solution but, instead, the combination of both  $(k_x/k_y)(H/L)^2$ , a discriminated dimensionless group.

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